UNSW THE UNIVERSITY OF NEW SOUTH WALES EDUCATIONAL ASSESSMENT AUSTRALIA

MISTAKES, MISUNDERSTANDINGS & MISCALCULATIONS

A LOOK AT STUDENT ERRORS IN ASMA [ICAS-M]

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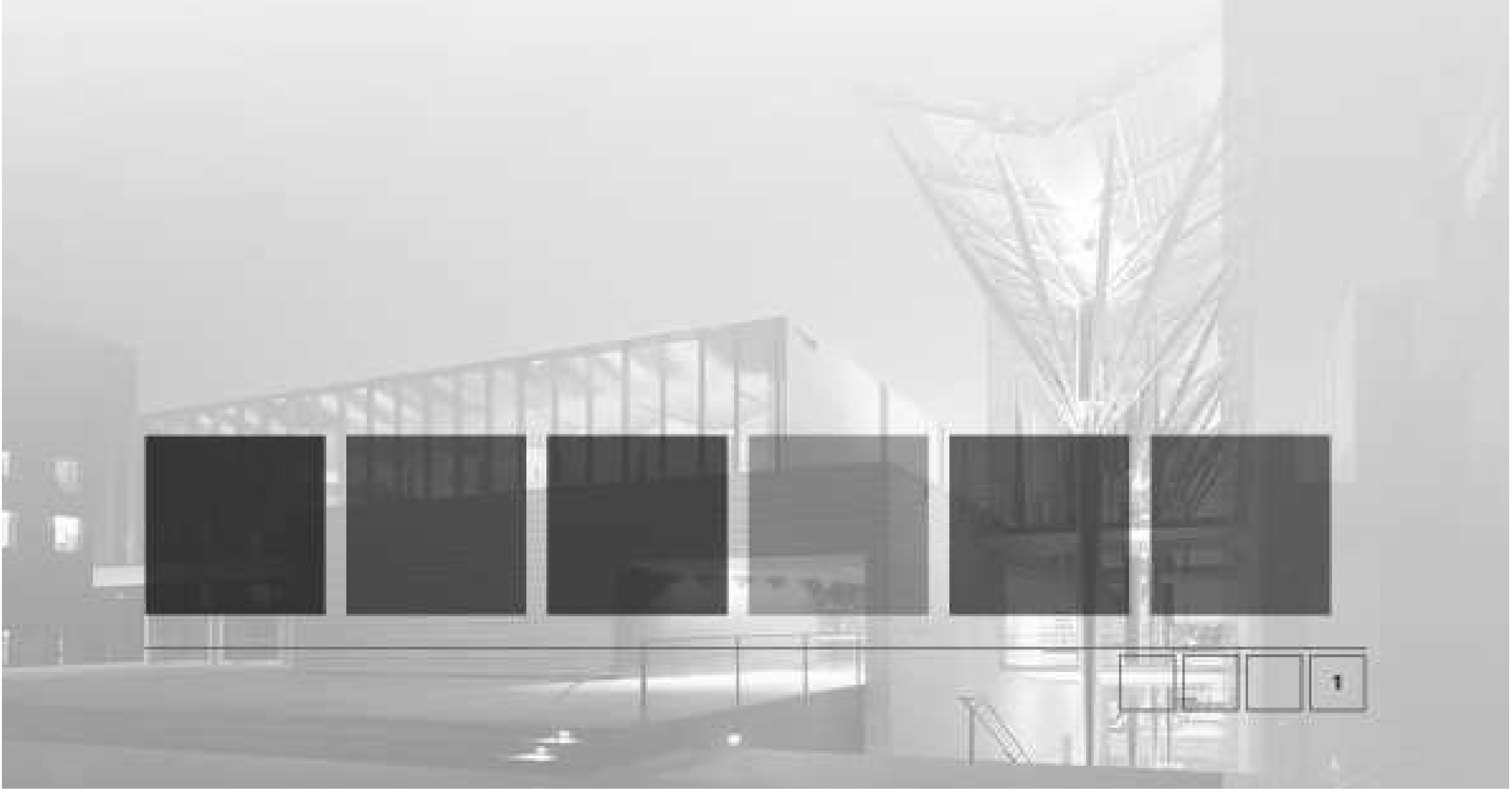


Content

Mistakes, misunderstandings and miscalculations in the Australasian Schools Mathematics Assessment (ASMA)

Nick Connolly October 2004

This article is based on some research I'm undertaking at Educational Assessment Australia at UNSW. It also formed the final part of my presentation at the marvellous MANSW Mudgee conference.

Background: What is ASMA? *[Now ICAS-M 2006]*

The Australasian Schools Mathematics Assessment is researched and produced by Educational Assessment Australia (formerly the Educational Testing Centre) a part of the University of New South Wales. ASMA has been running under this title since 2003, prior to this it had operated as the Primary Schools Mathematics Competition.

The assessment is under taken not only in Australia but also in New Zealand, Singapore, Brunei, South Africa and the Pacific Region. Versions of the test are also taken in Hong Kong, and most recently in India.

In New South Wales a wide range of different kinds of schools enter; public, private, home schools and distance education. Some schools enter a few students and use the test primarily as a competition; other schools enter whole cohorts and make additional use of the assessment aspects of ASMA

[*From 2006 ASMA was renamed International Competitions and Assessments for Schools – Mathematics, or ICAS-M*]

Make up of the Test

There are eight different papers: Year 3, Year 4, Year 5, Year 6, Year 7, Year 8, Years 9&10 and Years 11&12. The test consists of 40 items. In Year 6 and above the first thirty-five items are multiple choice and the last five are free response. All items are worth 1 mark and there is no negative marking. Complicated marking schemes and negative marking do not (in our opinion) generate any extra useful information about a student's ability.

Free Response Items

The last 5 Items are free response items. Each item has a numerical answer ranging from 0 to 999. The idea behind these items was to ensure that the assessment could carry very challenging items that were not also prone to guessing.

It is not necessarily the case that multiple choice items are prone to guessing. Well written items with plausible options will engage students sufficiently that they will attempt the item and also find that their response is one of the options available to them. However for items that are particularly challenging it is inevitable that a certain amount of guessing will occur. Free response items don't eliminate guessing (as we shall see) but they do make it very unlikely that student will guess the correct answer.

There is a significant downside to free response items. With multiple choice items it is relatively easy to analyse the kinds of incorrect answers (distractors) students are choosing. This kind of "distractor-analysis" can be very revealing about the kind of errors student make but is limited to only those errors that have been anticipated by the test constructor. If a student makes an error with a multiple choice item and the option is not available, then that error goes unrecorded and instead the student will have to choose some other option.

The free-response items in ASMA have the advantage that unlike multiple choice items students can record a much wider range of responses. However because all the students responses are scanned and recorded electronically it is still possible to analyse the kinds of responses students are giving.
By monitoring the frequency of student responses we can use the free-response items to help us develop better items, including multiple choice items, by having a better understanding of the kind of errors students make.

The non-response response

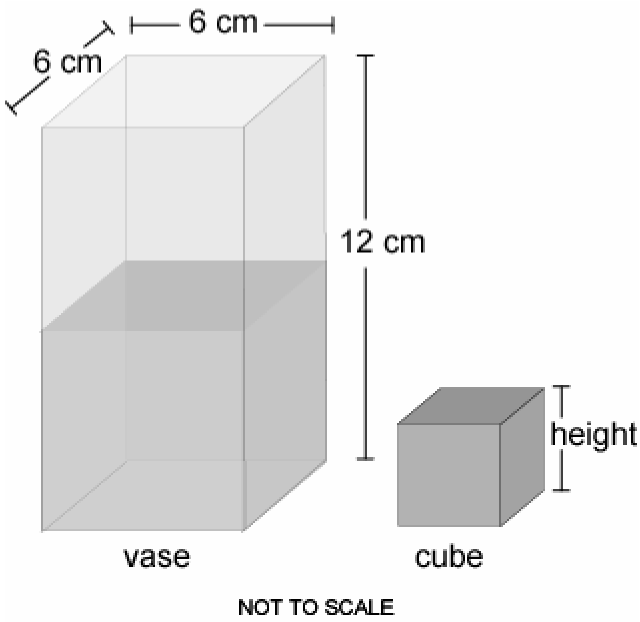
The free-response items in ASMA are both some of the hardest items in the papers and also the very last items. Consequently many students do not attempt all of them. Of the students who do attempt them not all put down entirely serious answers.
For each item the number “999” is chosen by a few students as is “0” and “000”. The frequency with which the “999” appears is completely independent of the nature of the question.
A more imaginative response from another tiny number of students is to answer the first free-response item with “123”, then “456”, and then “789”. At that point some give up but a few brave souls press on with “101”, and “112”!
These kinds of responses, where the student has apparently not fully engaged with the item, are few in number but appear consistently through the year groups. Year 9, however, produce a much broader range of novel answers: “WTF” an acronym for a profane expression of puzzlement; “666” has appeared a couple of times in 2003 and 2004; one year 9 student in NSW answered all the items with “69” and finally the stress of it all brought one student to answer with “POO” and “WEE”. Luckily a four-letter response is not available to students.

Some Responses ASMA 2004 Year 7

I have analysed some of the responses given by Year 7 students in New South Wales to the free response items in the ASMA 2004 paper. I will give a detailed analysis of one item to illustrate the process and some figures for another.

Question F1

Ray has a vase in the shape of a square prism.



He fills the vase half full of water.

After a cube was dropped into the vase, the volume of the water and the cube was 341 cm^3.

What is the **height**, in cm, of the cube?

About 3750 students in NSW attempted this question. Breaking the item down into a series of steps shows the kinds of errors that students might make. It also illustrates what options should be used if this item were made into a multiple choice item.

- First step: calculate the volume of the vase

  $6 \times 6 \times 12 = 432$

6 students answered 432 (0.16% of students who attempted the item)
5 students answered 36 (6×6)
9 students answered 72 (6×12)

- Second step: halve the volume

  $432 \div 2 = 216$

7 students answered 216

- Third step: calculate difference

341 – 216 = 125

144 students answered 125 (about 4% of those who attempted the item)

"125" was in the top 6 responses to the item and represents the first major error that a significant number of students made.

- Fourth step: find the cube root of 125 without a calculator

$5 \times 5 \times 5 = 125$

About 18% of those who attempted the question gave the correct answer 5 and it was the most popular answer. The numbers from 2 to 6 accounted for over 55% of students' answers which probably indicates students making use of the diagram and common sense to estimate answers or possibly failed attempts to find a number whose cube gives 125. Note that the item does not necessarily assume students are familiar with cube roots or the terminology of cube roots.

Other less obvious answers from students included numbers which seem to result from trying out common operations on the figures included in the question. The figure 24 was given in about 1% of responses (6 + 6 + 12?). Another 1% simply gave the figure 341 already mentioned in the item. Notably 35% of those who attempted the question gave the height of the cube as being **greater** than 12 cm, which is greater than the height (and width) of the vase it was put into!

Readers might want to also work out why ninety students gave 41 as answer (consider the last step…).

Question F3

Talia had two identical pies.

She cut the first pie into 6 equal pieces.

She cut the second pie into 8 equal pieces.

The mass of each piece of the first pie was 40 grams more than the mass of each piece of the second pie.

What was the mass of each pie, in grams?

This was a tricky problem of some depth and subtly. Students gave a range of answers with all of the most frequent answers being multiples of ten. The most common response (approximately 19% of those who attempted the item) was "240". The correct answer was only the third most frequent response.

The table shows the top 10 most frequent responses

Student Responses	Percentage Frequency
20	6.69%
30	4.77%
60	2.99%

80	3.95%
120	2.09%
240	18.56%
320	3.05%
360	3.70%
480	3.84%
960	6.27%

Some of the responses are fairly obvious in origin ($240 = 6 \times 40$, $320 = 8 \times 40$). Other responses are less clear.

Debugging the mathematical brain

The world of software development has brought us the notion of "debugging": analysing the seemingly bizarre or irrational behaviour of a piece of software to find the fundamental failure in the logic of the program. Continuing our research into how students end up giving implausible answers to complicated problems, is giving us some insight into the "bugs" in mathematical reasoning, and a fascinating insight into the way students think.

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